OCR
Oxford Cambridge and RSA

Accredited

A Level Further Mathematics A
Y545 Additional Pure Mathematics
Sample Question Paper Version 2

Date – Morning/Afternoon
Time allowed: 1 hour 30 minutes

You must have:
Printed Answer Booklet
Formulae A Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 A curve is given by $x = t^2 - 2\ln t$, $y = 4t$ for $t > 0$. When the arc of the curve between the points where $t = 1$ and $t = 4$ is rotated through 2π radians about the *x*-axis, a surface of revolution is formed with surface area *A*.

Given that $A = k\pi$, where *k* is an integer,

- write down an integral which gives *A* and
- find the value of *k*. **[4]**

2 Find the volume of tetrahedron OABC, where O is the origin, A = (2, 3, 1), B = (–4, 2, 5) and C = (1, 4, 4). **[3]**

3 Given $z = x\sin y + y\cos x$, show that $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} + z = 0$. **[5]**

4 **(i)** Solve the recurrence relation $u_{n+2} = 4u_{n+1} - 4u_n$ for $n \geq 0$, given that $u_0 = 1$ and $u_1 = 1$. **[4]**

(ii) Show that each term of the sequence $\{u_n\}$ is an integer. **[2]**

5 **In this question you must show detailed reasoning.**

It is given that $I_n = \int_0^{\pi} \sin^n \theta \, \mathrm{d}\theta$ for $n \geq 0$.

(i) Prove that $I_n = \dfrac{n-1}{n} I_{n-2}$ for $n \geq 2$. **[5]**

(ii) **(a)** Evaluate I_1. **[2]**

(b) Use the reduction formula to determine the exact value of $\int_0^{\pi} \cos^2 \theta \, \sin^5 \theta \, \mathrm{d}\theta$. **[2]**

6 A surface *S* has equation $z = \mathrm{f}(x, y)$, where $\mathrm{f}(x, y) = 2x^2 - y^2 + 3xy + 17y$. It is given that *S* has a single stationary point, *P*.

(i) **(a)** Determine the coordinates of *P*. **[5]**

(b) Determine the nature of *P*. **[3]**

(ii) Find the equation of the tangent plane to *S* at the point $Q(1, 2, 38)$. **[2]**

7 In order to rescue them from extinction, a particular species of ground-nesting birds is introduced into a nature reserve. The number of breeding pairs of these birds in the nature reserve, t years after their introduction, is an integer denoted by N_t. The initial number of breeding pairs is given by N_0.

An initial discrete population model is proposed for N_t.

$$\text{Model I: } N_{t+1} = \tfrac{6}{5} N_t \left(1 - \tfrac{1}{900} N_t\right)$$

(i) (a) For Model I, show that the steady state values of the number of breeding pairs are 0 and 150. **[3]**

(b) Show that $N_{t+1} - N_t < 150 - N_t$ when N_t lies between 0 and 150. **[3]**

(c) Hence find the long-term behaviour of the number of breeding pairs of this species of birds in the nature reserve predicted by Model I when $N_0 \in (0, 150)$. **[2]**

An alternative discrete population model is proposed for N_t.

$$\text{Model II: } N_{t+1} = \text{INT}\left(\tfrac{6}{5} N_t \left(1 - \tfrac{1}{900} N_t\right)\right)$$

(ii) (a) Given that $N_0 = 8$, find the value of N_4 for each of the two models. **[2]**

(b) Which of the two models gives values for N_t with the more appropriate level of precision? **[1]**

8 The set X consists of all 2×2 matrices of the form $\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$, where x and y are real numbers which are not **both** zero.

(i) (a) The matrices $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ and $\begin{pmatrix} c & -d \\ d & c \end{pmatrix}$ are both elements of X.

Show that $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}\begin{pmatrix} c & -d \\ d & c \end{pmatrix} = \begin{pmatrix} p & -q \\ q & p \end{pmatrix}$ for some real numbers p and q to be found in terms of a, b, c and d. **[2]**

(b) Prove by contradiction that p and q are not **both** zero. **[5]**

(ii) Prove that X, under matrix multiplication, forms a group G.
[You may use the result that matrix multiplication is associative.] **[4]**

(iii) Determine a subgroup of G of order 17. **[2]**

9 **(i) (a)** Prove that $p \equiv \pm 1 \pmod 6$ for all primes $p > 3$. **[2]**

(b) Hence or otherwise prove that $p^2 - 1 \equiv 0 \pmod{24}$ for all primes $p > 3$. **[3]**

(ii) Given that p is an odd prime, determine the residue of 2^{p^2-1} modulo p. **[4]**

(iii) Let p and q be distinct primes greater than 3. Prove that $p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}$. **[5]**

END OF QUESTION PAPER

OCR
Oxford Cambridge and RSA

…day June 20XX – Morning/Afternoon

A Level Further Mathematics A

Y545 Additional Pure

SAMPLE MARK SCHEME

Duration: 1 hour 30 minutes

MAXIMUM MARK 75

This document consists of 16 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Further Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some papers. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1			$\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2=\left(2t-\frac{2}{t}\right)^2+16$	M1 A1	1.1a 1.1	Correct unsimplified	May be seen in their expression for A.
			$A=2\pi\int_1^4 4t\sqrt{\left(2t-\frac{2}{t}\right)^2+16}\,dt$	M1	1.2	Use of Surface Area formula	Must be seen
			$=384\pi$ so $k=384$	A1 [4]	1.1	BC	
2			$V=\frac{1}{6}\mathbf{a}\cdot\mathbf{b}\times\mathbf{c}$	M1	1.1a	Attempt to use the triple scalar product	
			$=\frac{1}{6}\times\begin{pmatrix}2\\3\\1\end{pmatrix}\cdot\begin{pmatrix}-12\\21\\-18\end{pmatrix}=3.5$	A1 A1 FT [3]	1.1 1.1	A correct vector product BC BC FT their vector product if seen	
3			$\frac{\partial z}{\partial x}=\sin y-y\cos x$ $\quad\frac{\partial z}{\partial y}=x\cos y+\cos x$	M1 A1	2.1 1.1	Attempt at 1st and 2nd partial derivatives of z with respect to x and y; at least one correct	
			$\frac{\partial^2 z}{\partial x^2}=-y\cos x$ $\quad\frac{\partial^2 z}{\partial y^2}=-x\sin y$	A1 FT A1 FT	1.1 1.1	FT for each correct second partial derivative	
			$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}+z$ $=-y\cos x-x\sin y+(x\sin y+y\cos x)=0$	E1 [5]	2.1	AG Shown clearly	

Question			Answer	Marks	AO	Guidance	
4	(i)		Characteristic Equation is $\lambda^2 - 4\lambda + 4 = 0$ $\Rightarrow \lambda = 2$ (twice) and General Solution is $u_n = (A + Bn) \times 2^n$ $u_0 = 1$ and $u_1 = 1 \Rightarrow 1 = A$ and $1 = 2(A + B)$ $\Rightarrow A = 1, B = -\frac{1}{2}$ and $u_n = \left(1 - \frac{1}{2}n\right) \times 2^n$	**M1** **A1** **M1** **A1 FT** **[4]**	**1.1a** **1.1** **1.1** **1.1**	Substitution of first two known terms FT correctly solved sim. eqns.	
4	(ii)		So $u_n = \left(2 - n\right) \times 2^{n-1}$ and both parts of the product are integers so u_n is an integer	**M1** **E1** **[2]**	**3.1a** **2.4**		Do not penalise lack of checking for $u_0 = 1$ since this is given

Question			Answer	Marks	AO	Guidance	
5	(i)		**DR** $I_n = \int_0^{\pi} \sin\theta.\sin^{n-1}\theta \ \mathrm{d}\theta$	**M1**	**2.1**	Correct splitting and attempt at $\int n$ by parts	
			$= \left[-\cos\theta.\ \sin^{n-1}\theta \right]_0^{\pi} -$ $\ldots \int_0^{\pi} -\cos\theta.(n-1)\sin^{n-2}\theta.\cos\theta \ \mathrm{d}\theta$	**A1**	**1.1**		
			$= 0 + (n-1)\int_0^{\pi} \left(1-\sin^2\theta\right)\sin^{n-2}\theta \ \mathrm{d}\theta$	**M1**	**2.1**	Use of trigonometric identity in order to obtain …	
			$= (n-1)\{I_{n-2} - I_n\}$	**A1**	**1.1**	... substitution for *I*	
			$\Rightarrow I_n = \dfrac{n-1}{n} I_{n-2}$	**E1** **[5]**	**1.1**	AG	
5	(ii)	(a)	**DR** $I_1 = \int_0^{\pi} \sin\theta \, \mathrm{d}\theta$	**M1**	**3.1a**		
			$= 2$	**A1** **[2]**	**1.1**		
5	(ii)	(b)	**DR** Using $c^2 = 1 - s^2$, $I = I_5 - I_7$	**M1**	**3.1a**	Repeated use of RF soi	
			$I_5 - I_7 = \left(1 - \frac{6}{7}\right) I_5 = \frac{1}{7} \times \frac{4}{5} I_3$				
			$= \frac{1}{7} \times \frac{4}{5} \times \frac{2}{3} I_1 = \frac{16}{105}$	**E1** **[2]**	**2.1**	Correct answer with sufficient detail to show the reduction process	

Question			Answer	Marks	AO	Guidance	
6	(i)	(a)	$f_x = 4x + 3y$ and	B1	1.1		
			$f_y = -2y + 3x + 17$	B1	1.1		
			Both are zero when				
			$4x + 3y = 0$ and $-2y + 3x + 17 = 0$	M1	2.1	The connection must be made explicitly	
			$x = -3,\ y = 4$	A1	1.1	BC	
			$z = 34$	A1	1.1		
				[5]			
		(b)	$f_{xx} = 4,\ f_{yy} = -2,\ f_{xy} = f_{yx} = 3$	M1	2.1	Evaluating all second partial derivatives and attempt to use the Hessian	
			$\lvert\mathbf{H}\rvert = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} 4 & 3 \\ 3 & -2 \end{vmatrix} = -17$	A1	1.1	FT from their above values	
			$\lvert\mathbf{H}\rvert < 0 \Rightarrow$ saddle-point	E1	2.2a	Correct conclusion from a negative determinant value	
				[3]			
6	(ii)		$z = f(a, b) + (x - a)f_x(a, b)$ $+(y - b)f_y(a, b)$	M1	1.1	Quoting correct form, including attempt to substitute in values	
			i.e. $z = 38 + 10(x - 1) + 16(y - 2)$	A1	1.1	Any correct simplified form	
			or $10x + 16y - z = 4$				
				[2]			

Question			Answer	Marks	AO	Guidance	
7	(i)	(a)	$N_{t+1} - N_t = \frac{1}{5}N_t - \frac{1}{750}N_t^2$ or $\frac{1}{750}N_t(150 - N_t)$	B1	3.4		
			Let M be a steady state value, then $\frac{1}{750}M(150 - M) = 0$	M1	2.1		
			$M = 0$ or 150	A1	1.1		
				[3]			
7	(i)	(b)	$N_{t+1} - N_t = \frac{1}{750}N_t(150 - N_t)$	M1	1.1	soi	
			$N_t \in (0,150) \Rightarrow \frac{1}{750}N_t \in (0, \frac{1}{5})$	M1	2.1		
			Therefore, since $150 - N_t > 0$				
			We have that $N_{t+1} - N_t < 150 - N_t$	E1	2.2a		
				[3]			
7	(i)	(c)	N_t increases to approach 150 in the long term, without oscillation	B1	3.4	For "150"	
				B1	3.4	For "without oscillation" oe	
				[2]			
7	(ii)	(a)	For Model I $N_4 = 15.82...$	B1	1.1	BC Must show a non-integer answer	
			For Model II $N_4 = 13$	B1	1.1	BC	Using INT or by rounding after each iteration
				[2]			
7	(ii)	(b)	E.g. The number of breeding pairs must be an integer, so Model II is the more appropriate model because it gives only integer values.	E1	3.5a	For considering the appropriateness of the models given that the context requires N_t to be an integer.	
				[1]			

Question			Answer	Marks	AO	Guidance	
8	(i)	(a)	$\begin{pmatrix} a & -b \\ b & a \end{pmatrix}\begin{pmatrix} c & -d \\ d & c \end{pmatrix}=\begin{pmatrix} ac-bd & -(ad+bc) \\ ad+bc & ac-bd \end{pmatrix}$	M1	1.1		
			i.e. $p=ac-bd$ and $q=ad+bc$	A1 [2]	1.1	May be clear from the product matrix	
8	(i)	(b)	Suppose that both $p=0$ and $q=0$, so that $ac-bd=0$ and $bc+ad=0$	B1	1.1	Explicit assumption that both are zero Condition that *a* and *b* not both zero and *c* and *d* not both zero need not be explicitly stated	**OR** **B1** Suppose that both $p=0$ and $q=0$ so that $\det\begin{pmatrix} p & -q \\ q & p \end{pmatrix}=p^2+q^2=0$
			Then $acd-bd^2=0$ and $acd+bc^2=0$ Subtracting $\Rightarrow b\left(c^2+d^2\right)=0$	M1	3.1a	e.g. treating as simultaneous equations (either *a*, *b* or *c*, *d*)	**M1** Then $\det\left[\begin{pmatrix} a & -b \\ b & a \end{pmatrix}\begin{pmatrix} c & -d \\ d & c \end{pmatrix}\right]=0$
			Since *c* and *d* are not both zero $c^2+d^2\neq 0$ $\Rightarrow b=0$	A1	2.2a	One case correctly concluded	**A1** so that $c^2+d^2=0$ and/or $a^2+b^2=0$
			Similarly, $ac-bd=0$ and $bc+ad=0\Rightarrow a\left(c^2+d^2\right)=0$ Since *c* and *d* are not both zero $c^2+d^2\neq 0$ $\Rightarrow a=0$	A1	2.2a	Second case correctly concluded	**A1** since neither {*c* and *d*} or {*a* and *b*} can both be zero…
			This contradicts the condition that *a*, *b* are not both zero, so the assumption that *p* and *q* are both zero is false.	E1 [5]	2.4	Contradiction clearly explained	**E1** This contradicts the condition that *a*, *b* and *c*, *d* are not both zero, so the assumption that *p* and *q* are both zero is false.

Question			Answer	Marks	AO	Guidance	
8	(ii)		Closure follows from part **(i)** (Associativity is given) Identity is when $a=1,\ b=0$ or noting $\begin{pmatrix}1 & (-)0\\0 & 1\end{pmatrix}\in X$ Inverse of $\begin{pmatrix}a & -b\\b & a\end{pmatrix}$ is $\frac{1}{a^2+b^2}\begin{pmatrix}a & -(-b)\\-b & a\end{pmatrix}\in X$ since $a^2+b^2\neq 0$ (as a, b not both zero)	**B1** **B1** **B1** **E1** **[4]**	**1.1** **1.1** **2.1** **2.3**	To earn this, they must show appropriate form for $\mathbf{A}^{-1}$ and note why it exists	
8	(iii)		The matrix with $a=\cos\left(\frac{2}{17}\pi\right),\ b=\sin\left(\frac{2}{17}\pi\right)$ generates … the subgroup of rotations about O (through multiples of $\frac{2}{17}\pi$)	**B1** **B1** **[2]**	**3.1a** **2.4**	Suitable values of a and b Description of the subgroup	
9	(i)	(a)	If $p\equiv 0$ or 3 (mod 6) then it is a multiple of 3 If $p\equiv 0$, 2 or 4 (mod 6) then it is even Hence $p\equiv\pm 1$ (mod 6)	**B1** **E1** **[2]**	**2.1** **1.1**	The case $p\equiv 0$ needs only be dealt with once	May be in the form $p=6k+r$ ($r=0$ to 5)

Question			Answer	Marks	AO	Guidance	
9	(i)	(b)	$p^2-1=(6k\pm1)^2-1=36k^2\pm12k$ $=12k(3k\pm1)$ If k is even then $24\mid12k$ If k is odd then $3k\pm1$ is even and $24\mid12k(3k\pm1)$	**M1** **A1** **E1** **[3]**	**2.1** **1.1** **2.4**	Must be in a suitable form to make following deductions All fully explained	**OR** **M1A1** Calculate all squares mod 24 (**M1** if no more than 2 independent errors) **E1** Explain that those congruent to 1 are $\pm1, \pm5, \pm7, \pm11$ i.e. all those $\equiv\pm1\pmod6$, of which the primes are a subset
9	(ii)		$2^{p^2-1}=2^{(p-1)(p+1)}$ $=\left(2^{p-1}\right)^{p+1}$ $\equiv(1)^{p+1}\pmod p$ $=1$	**M1** **M1** **E1** **A1** **[4]**	**3.1a** **3.1a** **2.1** **3.2a**	 By Fermat's little theorem, since hcf $(2,\ p)=1$	
9	(iii)		$p^{q-1}\equiv1\pmod q$ $q^{p-1}\equiv0\pmod q$ $\Rightarrow p^{q-1}+q^{p-1}\equiv1\pmod q$ Similarly, $p^{q-1}+q^{p-1}\equiv1\pmod p$ Together, $p^{q-1}+q^{p-1}\equiv1\pmod{pq}$	**B1** **B1** **B1** **B1** **E1** **[5]**	**1.1** **1.1** **3.1a** **3.1a** **2.2a**	By FLT (first result, either way round) Second result Since hcf $(p,\ q)=1$	 Must be justified

Assessment Objectives (AO) Grid

Question	AO1	AO2	AO3(PS)	AO3(M)	Total
1	4	0	0	0	4
2	3	0	0	0	3
3	3	2	0	0	5
4(i)	4	0	0	0	4
4(ii)	0	1	1	0	2
5(i)	3	2	0	0	5
5(ii)(a)	1	0	1	0	2
5(ii)(b)	0	1	1	0	2
6(i)(a)	4	1	0	0	5
6(i)(b)	1	2	0	0	3
6(ii)	2	0	0	0	2
7(i)(a)	1	1	0	1	3
7(i)(b)	1	2	0	0	3
7(i)(c)	0	0	0	2	2
7(ii)(a)	2	0	0	0	2
7(ii)(b)	0	0	0	1	1
8(i)(a)	2	0	0	0	2
8(i)(b)	1	3	1	0	5
8(ii)	2	2	0	0	4
8(iii)	0	1	1	0	2
9(i)(a)	1	1	0	0	2
9(i)(b)	1	2	0	0	3
9(ii)	0	1	3	0	4
9(iii)	2	1	2	0	5
Totals	38	23	10	4	75

PS = Problem Solving
M = Modelling

Summary of Updates

Date	Version	Change
October 2019	2	Amendments to the front cover rubric instructions to candidates

BLANK PAGE

BLANK PAGE

OCR
Oxford Cambridge and RSA

Accredited

A Level Further Mathematics A
Y545 Additional Pure Mathematics
Printed Answer Booklet Version 2

Date – Morning/Afternoon
Time allowed: 1 hour 30 minutes

You must have:
- Question Paper Y545 (inserted)
- Formulae A Level Further Mathematics A

You may use:
- a scientific or graphical calculator

First name	
Last name	

Centre number						**Candidate number**				

INSTRUCTIONS
- The Question Paper will be found inside the Printed Answer Booklet.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$

INFORMATION
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

1

2

3	

4(i)	
4(ii)	

5(i)	
5(ii)(a)	
5(ii)(b)	

6(i)(a)	
6(i)(b)	
6(ii)	

7(i)(a)	
7(i)(b)	
7(i)(c)	

7(ii)(a)	
7(ii)(b)	

DO NOT WRITE IN THIS SPACE

8(i)(a)

8(i)(b)

8(ii)

8(iii)

9(i)(a)	
9(i)(b)	
9(ii)	

9(iii)	