

A Level Further Mathematics A

Y545 Additional Pure Mathematics

Sample Question Paper

Version 2

Date – Morning/Afternoon

Time allowed: 1 hour 30 minutes

You must have:

Printed Answer Booklet
Formulae A Level Further Mathematics A

You may use:

- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

- 1 A curve is given by $x = t^2 - 2\ln t$, $y = 4t$ for $t > 0$. When the arc of the curve between the points where $t = 1$ and $t = 4$ is rotated through 2π radians about the x -axis, a surface of revolution is formed with surface area A .
Given that $A = k\pi$, where k is an integer,
- write down an integral which gives A and
 - find the value of k . [4]
- 2 Find the volume of tetrahedron OABC, where O is the origin, $A = (2, 3, 1)$, $B = (-4, 2, 5)$ and $C = (1, 4, 4)$. [3]
- 3 Given $z = x\sin y + y\cos x$, show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} + z = 0$. [5]
- 4 (i) Solve the recurrence relation $u_{n+2} = 4u_{n+1} - 4u_n$ for $n \geq 0$, given that $u_0 = 1$ and $u_1 = 1$. [4]
- (ii) Show that each term of the sequence $\{u_n\}$ is an integer. [2]
- 5 **In this question you must show detailed reasoning.**
It is given that $I_n = \int_0^\pi \sin^n \theta d\theta$ for $n \geq 0$.
- (i) Prove that $I_n = \frac{n-1}{n} I_{n-2}$ for $n \geq 2$. [5]
- (ii) (a) Evaluate I_1 . [2]
- (b) Use the reduction formula to determine the exact value of $\int_0^\pi \cos^2 \theta \sin^5 \theta d\theta$. [2]
- 6 A surface S has equation $z = f(x, y)$, where $f(x, y) = 2x^2 - y^2 + 3xy + 17y$. It is given that S has a single stationary point, P .
- (i) (a) Determine the coordinates of P . [5]
- (b) Determine the nature of P . [3]
- (ii) Find the equation of the tangent plane to S at the point $Q(1, 2, 38)$. [2]

- 7 In order to rescue them from extinction, a particular species of ground-nesting birds is introduced into a nature reserve. The number of breeding pairs of these birds in the nature reserve, t years after their introduction, is an integer denoted by N_t . The initial number of breeding pairs is given by N_0 .

An initial discrete population model is proposed for N_t .

$$\text{Model I: } N_{t+1} = \frac{6}{5} N_t \left(1 - \frac{1}{900} N_t\right)$$

- (i) (a) For Model I, show that the steady state values of the number of breeding pairs are 0 and 150. [3]
- (b) Show that $N_{t+1} - N_t < 150 - N_t$ when N_t lies between 0 and 150. [3]
- (c) Hence find the long-term behaviour of the number of breeding pairs of this species of birds in the nature reserve predicted by Model I when $N_0 \in (0, 150)$. [2]

An alternative discrete population model is proposed for N_t .

$$\text{Model II: } N_{t+1} = \text{INT}\left(\frac{6}{5} N_t \left(1 - \frac{1}{900} N_t\right)\right)$$

- (ii) (a) Given that $N_0 = 8$, find the value of N_4 for each of the two models. [2]
- (b) Which of the two models gives values for N_t with the more appropriate level of precision? [1]

- 8 The set X consists of all 2×2 matrices of the form $\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$, where x and y are real numbers which are not **both** zero.

(i) (a) The matrices $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ and $\begin{pmatrix} c & -d \\ d & c \end{pmatrix}$ are both elements of X .

Show that $\begin{pmatrix} a & -b \\ b & a \end{pmatrix} \begin{pmatrix} c & -d \\ d & c \end{pmatrix} = \begin{pmatrix} p & -q \\ q & p \end{pmatrix}$ for some real numbers p and q to be found in terms of a, b, c and d . [2]

(b) Prove by contradiction that p and q are not **both** zero. [5]

(ii) Prove that X , under matrix multiplication, forms a group G . [4]
[You may use the result that matrix multiplication is associative.]

(iii) Determine a subgroup of G of order 17. [2]

- 9 (i) (a) Prove that $p \equiv \pm 1 \pmod{6}$ for all primes $p > 3$. [2]

(b) Hence or otherwise prove that $p^2 - 1 \equiv 0 \pmod{24}$ for all primes $p > 3$. [3]

(ii) Given that p is an odd prime, determine the residue of 2^{p^2-1} modulo p . [4]

(iii) Let p and q be distinct primes greater than 3. Prove that $p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}$. [5]

END OF QUESTION PAPER

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