

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Sample Question Paper

Date – Morning/Afternoon

Version 2.1

Time allowed: 2 hours

You must have:

- Printed Answer Booklet

You may use:

- a scientific or graphical calculator



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INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

Formulae

A Level Mathematics A (H240)

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$$\sin \theta \approx \theta, \cos \theta \approx 1 - \frac{1}{2}\theta^2, \tan \theta \approx \theta \text{ where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2} h \{ (y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}) \}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B | A) = P(B)P(A | B) \quad \text{or} \quad P(A | B) = \frac{P(A \cap B)}{P(B)}$$

Standard deviation

$$\sqrt{\frac{\sum (x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Section A: Pure MathematicsAnswer **all** the questions

- 1 **(a)** If $|x| = 3$, find the possible values of $|2x - 1|$. **[3]**
- (b)** Find the set of values of x for which $|2x - 1| > x + 1$.
 Give your answer in set notation. **[4]**
- 2 **(a)** Use the trapezium rule, with four strips each of width 0.25, to find an approximate value for $\int_0^1 \frac{1}{\sqrt{1+x^2}} dx$. **[3]**
- (b)** Explain how the trapezium rule might be used to give a better approximation to the integral given in part **(a)**. **[1]**
- 3 **In this question you must show detailed reasoning.**
- Given that $5\sin 2x = 3\cos x$, where $0^\circ < x < 90^\circ$, find the exact value of $\sin x$. **[4]**
- 4 For a small angle θ , where θ is in radians, show that $1 + \cos \theta - 3\cos^2 \theta \approx -1 + \frac{5}{2}\theta^2$. **[4]**

5 (a) Find the first three terms in the expansion of $(1+px)^{\frac{1}{3}}$ in ascending powers of x . [3]

(b) The expansion of $(1+qx)(1+px)^{\frac{1}{3}}$ is $1+x-\frac{2}{9}x^2+\dots$.

Find the possible values of the constants p and q . [5]

6 A curve has equation $y = x^2 + kx - 4x^{-1}$ where k is a constant.

Given that the curve has a minimum point when $x = -2$

- find the value of k
- show that the curve has a point of inflection which is not a stationary point. [7]

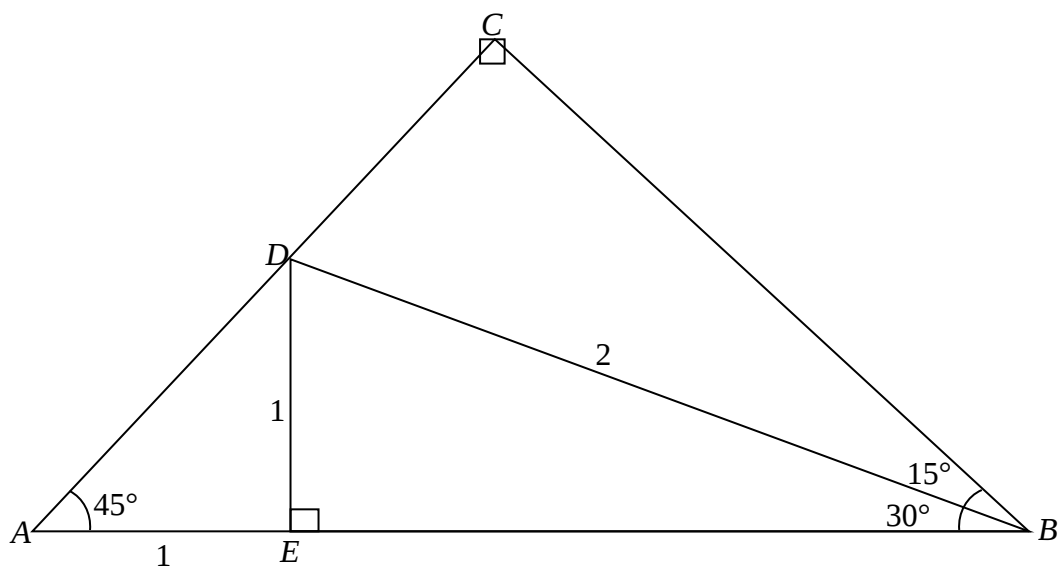
7 (a) Find $\int 5x^3\sqrt{x^2+1} \, dx$. [5]

(b) Find $\int \theta \tan^2 \theta \, d\theta$.

You may use the result $\int \tan \theta \, d\theta = \ln|\sec \theta| + c$. [5]

8 In this question you must show detailed reasoning.

The diagram shows triangle ABC .



The angles CAB and ABC are each 45° , and angle $ACB = 90^\circ$.

The points D and E lie on AC and AB respectively. $AE = DE = 1$, $DB = 2$.

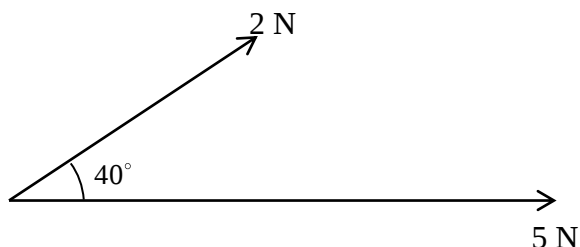
Angle $BED = 90^\circ$, angle $EBD = 30^\circ$ and angle $DBC = 15^\circ$.

(a) Show that $BC = \frac{\sqrt{2} + \sqrt{6}}{2}$. [3]

(b) By considering triangle BCD , show that $\sin 15^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$. [3]

Section B: Mechanics
Answer **all** the questions

- 9** Two forces, of magnitudes 2 N and 5 N, act on a particle in the directions shown in the diagram below.



- (a) Calculate the magnitude of the resultant force on the particle. [3]
- (b) Calculate the angle between this resultant force and the force of magnitude 5 N. [1]
- 10** A body of mass 20 kg is on a rough plane inclined at angle α to the horizontal. The body is held at rest on the plane by the action of a force of magnitude P N. The force is acting up the plane in a direction parallel to a line of greatest slope of the plane. The coefficient of friction between the body and the plane is μ .
- (a) When $P = 100$, the body is on the point of sliding down the plane.
- Show that $g \sin \alpha = g\mu \cos \alpha + 5$. [4]
- (b) When P is increased to 150, the body is on the point of sliding up the plane.
- Use this, and your answer to part (a), to find an expression for α in terms of g . [3]

- 11** In this question the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in the directions east and north respectively.

A particle of mass 0.12 kg is moving so that its position vector $\mathbf{r}$ metres at time t seconds is given by $\mathbf{r} = 2t^3\mathbf{i} + (5t^2 - 4t)\mathbf{j}$.

- (a) Show that when $t = 0.7$ the bearing on which the particle is moving is approximately 044° . **[3]**
- (b) Find the magnitude of the resultant force acting on the particle at the instant when $t = 0.7$. **[4]**
- (c) Determine the times at which the particle is moving on a bearing of 045° . **[2]**

12 A girl is practising netball.

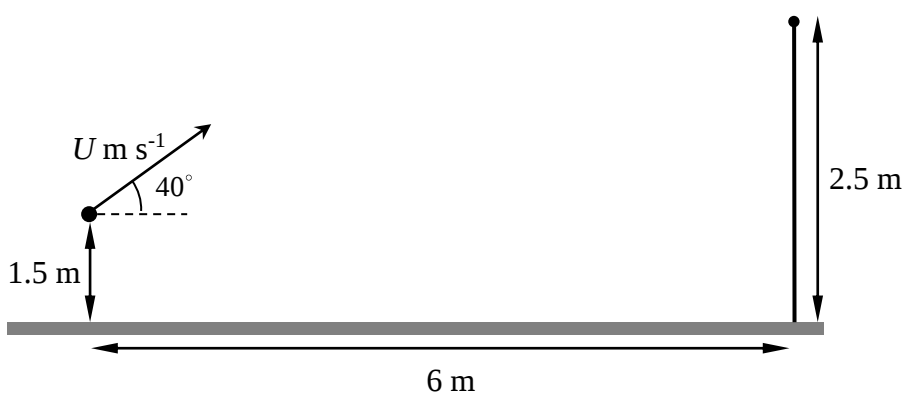
She throws the ball from a height of 1.5 m above horizontal ground and aims to get the ball through a hoop.

The hoop is 2.5 m vertically above the ground and is 6 m horizontally from the point of projection.

The situation is modelled as follows.

- The initial velocity of the ball has magnitude $U \text{ m s}^{-1}$.
- The angle of projection is 40° .
- The ball is modelled as a particle.
- The hoop is modelled as a point.

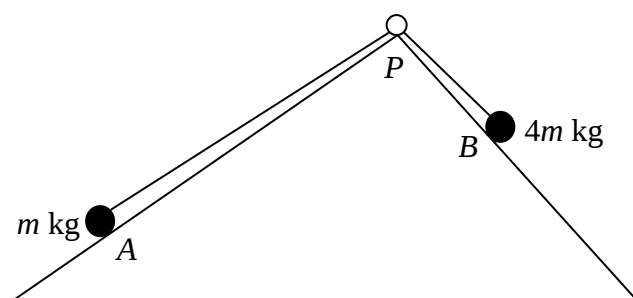
This is shown on the diagram below.



- (a) For $U = 10$, find
- the greatest height above the ground reached by the ball [5]
 - the distance between the ball and the hoop when the ball is vertically above the hoop. [4]
- (b) Calculate the value of U which allows her to hit the hoop. [3]
- (c) How appropriate is this model for predicting the path of the ball when it is thrown by the girl? [1]
- (d) Suggest one improvement that might be made to this model. [1]

- 13** Particle A, of mass m kg, lies on the plane Π_1 inclined at an angle of $\tan^{-1} \frac{3}{4}$ to the horizontal. Particle B, of $4m$ kg, lies on the plane Π_2 inclined at an angle of $\tan^{-1} \frac{4}{3}$ to the horizontal. The particles are attached to the ends of a light inextensible string which passes over a smooth pulley at P . The coefficient of friction between particle A and Π_1 is $\frac{1}{3}$ and plane Π_2 is smooth. Particle A is initially held at rest such that the string is taut and lies in a line of greatest slope of each plane.

This is shown on the diagram below.



- (a) Show that when A is released it accelerates towards the pulley at $\frac{7g}{15} \text{ m s}^{-2}$. [6]
- (b) Assuming that A does not reach the pulley, show that it has moved a distance of $\frac{1}{4}$ m when its speed is $\sqrt{\frac{7g}{30}} \text{ m s}^{-1}$. [2]
- 14** A uniform ladder AB of mass 35 kg and length 7 m rests with its end A on rough horizontal ground and its end B against a rough vertical wall. The ladder is inclined at an angle of 45° to the horizontal. A man of mass 70 kg is standing on the ladder at a point C, which is x metres from A. The coefficient of friction between the ladder and the wall is $\frac{1}{3}$ and the coefficient of friction between the ladder and the ground is $\frac{1}{2}$. The system is in limiting equilibrium.
- Find x . [8]

END OF QUESTION PAPER

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