

AS Level Further Mathematics A

Y535 Additional Pure Mathematics

Sample Question Paper

Version 2

Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

You must have:

- Printed Answer Book
- Formulae AS Level Further Mathematics A

You may use:

- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

- 1** The sequence $\{u_n\}$ is defined by $u_1 = 2$ and $u_{n+1} = \frac{12}{1+u_n}$ for $n \geq 1$.
Given that the sequence converges, with limit α , determine the value of α . **[3]**
- 2** The points $A(1, 2, 2)$, $B(8, 2, 5)$, $C(-3, 6, 5)$ and $D(-10, 6, 2)$ are the vertices of parallelogram $ABCD$.
Determine the area of $ABCD$. **[5]**
- 3** A non-commutative group G consists of the six elements $\{e, a, a^2, b, ab, ba\}$ where e is the identity element, a is an element of order 3 and b is an element of order 2.
By considering the row in G 's group table in which each of the above elements is pre-multiplied by b , show that $ba^2 = ab$. **[5]**
- 4** Let S be the set $\{16, 36, 56, 76, 96\}$ and $\times_H$ the operation of multiplication modulo 100.
- (i) Given that a and b are odd positive integers, show that $(10a+6)(10b+6)$ can also be written in the form $10n+6$ for some odd positive integer n . **[3]**
- (ii) Construct the Cayley table for $(S, \times_H)$ **[2]**
- (iii) Show that $(S, \times_H)$ is a group.
[You may use the result that $\times_H$ is associative on S .] **[3]**
- (iv) Write down all generators of $(S, \times_H)$. **[1]**
- 5** Let $f(x, y) = x^3 + y^3 - 2xy + 1$. The surface S has equation $z = f(x, y)$.
- (i) (a) Find f_x . **[1]**
- (b) Find f_y . **[1]**
- (c) Show that S has a stationary point at $(0, 0, 1)$. **[5]**
- (d) Find the coordinates of the second stationary point of S . **[1]**
- (ii) The section $z = f(a, y)$, where a is a constant, has exactly one stationary point.
Determine the equation of the section. **[3]**

- 6 A customer takes out a loan of $\pounds P$ from a bank at an annual interest rate of 4.9%. Interest is charged monthly at an equivalent monthly interest rate. This interest is added to the outstanding amount of the loan at the end of each month, and then the customer makes a fixed monthly payment of $\pounds M$ in order to reduce the outstanding amount of the loan.

Let L_n denote the outstanding amount of the loan at the end of month n **after** the fixed payment has been made, with $L_0 = P$.

- (i) Explain how the outstanding amount of the loan from one month to the next is modelled by the recurrence relation

$$L_{n+1} = 1.004L_n - M \quad (*)$$

with $L_0 = P$, $n \geq 0$.

[3]

- (ii) Solve, in terms of n , M and P , the first order recurrence relation given in part (i). [6]

- (iii) The loan amount is $\pounds 100\,000$ and will be fully repaid after 10 years. Find, to the nearest pound, the value of the monthly repayment. [3]

- (iv) The bank's procedures only allow for calculations using integer amounts of pounds. When each monthly amount of the outstanding debt (L_n) is calculated it is always rounded **up** to the nearest pound before the monthly repayment (M) is subtracted. Rewrite (*) to take this into account. [2]

- 7 (i) Let $N = 10a + b$ and $M = a - 5b$ where a and b are integers such that $a \geq 1$ and $0 \leq b \leq 9$. N is to be tested for divisibility by 17.

- (a) Prove that $17 \mid N$ if and only if $17 \mid M$. [5]

- (b) Demonstrate step-by-step how an algorithm based on these forms can be used to show that $17 \mid 4097$. [2]

- (ii) (a) Show that, for $n \geq 2$, any number of the form 1001_n is composite. [3]

- (b) Given that n is a positive even number, provide a counter-example to show that the statement "any number of the form 10001_n is prime" is false. [3]

END OF QUESTION PAPER

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...day June 20XX – Morning/Afternoon

AS Level Further Mathematics A

Y535 Additional Pure Mathematics

SAMPLE MARK SCHEME

Duration: 1 hour 15 minutes

MAXIMUM MARK 60



This document consists of 12 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for AS Level Further Mathematics A

- a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.
- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.
- g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some papers. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
- i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1			Set $u_{n+1} = u_n = \alpha \Rightarrow \alpha = \frac{12}{1+\alpha}$	M1	1.1	Use of the limit in the given recurrence relation	OR BC M1 “2” ENTER and iterating with 12 / (1 + ANS) A1 $\rightarrow \alpha = 3$
			$\alpha = 3$ and/or $\alpha = -4$	A1	1.1	BC Solving for α	
			Since $\alpha > 0$ (all terms are positive), $\alpha = 3$	E1	2.2a	Explicitly rejecting the negative solution or justifying why there is only one positive solution	
				[3]			
2			Area = $ (\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a}) $	M1	1.2	Use of formula with attempted substitution	BC OR using formula M1A1FT for $\begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - b_1a_3 \\ a_1b_2 - a_2b_1 \end{pmatrix} = \begin{pmatrix} -12 \\ -33 \\ 28 \end{pmatrix}$
			$= \left \begin{pmatrix} 7 \\ 0 \\ 3 \end{pmatrix} \times \begin{pmatrix} -4 \\ 4 \\ 3 \end{pmatrix} \right $	A1	1.1	Two correct vectors	
			$= \left \begin{pmatrix} -12 \\ -33 \\ 28 \end{pmatrix} \right $	M1 A1FT	1.1a 1.1	Attempt at vector product soi For vector product FT their vectors	
			$= \sqrt{2017} (= 44.9...)$	B1FT	1.1	FT their vector product	
				[5]			
3			$bG = \{be, ba, ba^2, b^2, bab, b^2a\}$	M1	1.1a	Attempt to pre-multiply by b	
			$= \{b, ba, ??, e, ??, a\}$ using $b^2 = e$	A1	1.1	All six elements, unsimplified	
				A1	1.1	Noting that four elements are known	
			So ba^2 is either a^2 or ab But $ba^2 = a^2$ only if $b = e$, which it isn't so $ba^2 = ab$	M1 E1	2.1 2.2a	Use of the Latin-Square property	
				[5]			

Question			Answer	Marks	AO	Guidance																																					
4	(i)		$=100ab + 60(a + b) + 36$ $=10(10ab + 6(a + b) + 3) + 6$ and $n = 10ab + 6(a + b) + 3$ is odd since it is even + even + odd	M1 A1 E1 [3]	1.1 3.1a 2.4	Correctly multiplied out given expression Correctly identified “tens” term Proper explanation that n is odd																																					
4	(ii)		<table><tr><td></td><td>16</td><td>36</td><td>56</td><td>76</td><td>96</td></tr><tr><td>16</td><td>56</td><td>76</td><td>96</td><td>16</td><td>36</td></tr><tr><td>36</td><td>76</td><td>96</td><td>16</td><td>36</td><td>56</td></tr><tr><td>56</td><td>96</td><td>16</td><td>36</td><td>56</td><td>76</td></tr><tr><td>76</td><td>16</td><td>36</td><td>56</td><td>76</td><td>96</td></tr><tr><td>96</td><td>36</td><td>56</td><td>76</td><td>96</td><td>16</td></tr></table>		16	36	56	76	96	16	56	76	96	16	36	36	76	96	16	36	56	56	96	16	36	56	76	76	16	36	56	76	96	96	36	56	76	96	16	M1 A1 [2]	1.1 1.1	For most entries correct (≤ 2 errors) For all entries correct	
	16	36	56	76	96																																						
16	56	76	96	16	36																																						
36	76	96	16	36	56																																						
56	96	16	36	56	76																																						
76	16	36	56	76	96																																						
96	36	56	76	96	16																																						
4	(iii)		Closure noted – no new elements in the table (Associativity given) 76 is the identity Inverse-pairs 16, 36 and 56, 96 noted	B1 B1 B1 [3]	2.5 1.2 1.1	Or from (i)																																					
4	(iv)		16, 36, 56, 96 are all possible generators	B1 [1]	2.2a																																						

Question			Answer	Marks	AO	Guidance	
6	(i)		R given by $^{12}\sqrt{1.049} = 1.004$ The amount owed at end of next month is $R \times$ amount owed at end of previous month $-M$ is his monthly repayment	B1 E1 E1 [3]	2.3 3.3 1.1		
6	(ii)		$L_{n+1} - RL_n = -M$ has Complementary Solution $L_n = AR^n$ For Particular Solution, try $L_n = b$ and substitute it into the recurrence relation to get $b(R-1) = M \Rightarrow b = 250M$ General Solution is thus $L_n = AR^n + 250M$ $L_0 = P \Rightarrow$ $A = P - 250M$ $\Rightarrow$ Solution is $L_n = (P - 250M) \times 1.004^n + 250M$	B1 M1 A1 B1FT M1 A1 [6]	1.2 1.1a 1.1 1.1 1.1 1.1	$(R=1.004)$ FT provided CS has 1 arbitrary constant and PS has none Use of known term to evaluate the constant	
6	(iii)		$L_n = 0$ when $n=120$ and $P = 100\,000$ gives $(100000 - 250M) \times 1.004^{120} + 250M = 0$ $\Rightarrow M = 1051$ (to nearest £)	M1 A1FT A1 [3]	3.1b 1.1 3.4	Substituting $L_n = 0$ and a numerical value of n into their solution Correct, unsimplified FT their n	85909.87239...

Question			Answer	Marks	AO	Guidance	
6	(iv)		(*) becomes $L_{n+1} = \text{INT}(1.004L_n + 1) - M$	M1 A1 [1]	3.5c 3.3	Expression involving L_n inside the INT function Correct Also $L_{n+1} = \text{INT}(1.004L_n - M + 1)$ or $L_{n+1} = \text{INT}(1.004L_n) - M + 1$	Accept equivalent “floor” or “ceiling” function expressions.
7	(i)	(a)	Let $17 \mid M$, then $M = 17m$ $\Rightarrow a - 5b = 17m$ $N = 10a + b = 10(a - 5b) + 51b$ $= 17m + 51b = 17(m + 3b)$ so $17 \mid N$ Let $17 \mid N$, then $N = 17n$ $\Rightarrow 10a + b = 17n$ $10M = 10a - 50b = (10a + b) - 51b$ $= 17n - 51b = 17(n - 3b)$ $\text{hcf}(10, 17) = 1$ so $17 \mid M$ therefore $17 \mid N$ if and only if $17 \mid M$	M1 A1 M1 A1 E1	1.1a 1.1 2.1 2.4 2.2a	Attempt either “if” or “only if” Simple case Attempt other direction Allow without $\text{hcf}(10, 17)$ considered $\text{hcf}(10, 17)$ oe considered and conclusion	OR M1 Consider $5N + M = 51a$ (or any $xN + yM = 17z$) M1 If $17 \mid N$, say $N = 17n$, Then $5N = 35n$ A1 so $M = 51a - 5N = 17(3a - 5n)$ and $17 \mid M$ A1 If $17 \mid M$, say $M = 17m$, then $5N = 51a - M = 17(3a - m)$ E1 $\text{hcf}(5, 17) = 1$ we have $17 \mid N$ therefore $17 \mid N$ if and only if $17 \mid M$
7	(i)	(b)	$4097 \rightarrow a = 409, b = 7$ $\rightarrow M = 409 - 35 = 374$ $374 \rightarrow a = 37, b = 4 \rightarrow M = 37 - 20 = 17$ Then $17 \mid 17 \Rightarrow 17 \mid 4097$	M1 A1 [2]	1.1 2.2a	Starting this process (first stage correctly attempted)	Including all working

Question			Answer	Marks	AO	Guidance	
7	(ii)	(a)	$1001_n = n^3 + 1$ $\equiv (n+1)(n^2 - n + 1)$ For $n \geq 2$ we have $n+1 \geq 3$... and $n^2 - n + 1 \geq 3$, so neither factor is 1 So 1001_n is composite	M1 M1 E1 [3]	1.1 2.1 2.4	Express as a polynomial in n Factorise and establish one factor as non-trivial Establish the other factor as non-trivial and conclude	
7	(ii)	(b)	$10001_n = n^4 + 1$ $n = 2 \quad 4 \quad 6 \quad 8$ $n^4 + 1 = 17 \quad 257 \quad 1297 \quad 4097 = 17k$ 17 is a factor of 4097 (from (i)(b)) so $n = 8$ provides a counter-example	B1 M1 E1 [3]	1.1 1.1 2.1	Method for searching for possible candidates NB $10^4 + 1 = 73 \times 137$, $12^4 + 1 = 89 \times 233$, $14^4 + 1 = 41 \times 937$, $16^4 + 1$ is prime, etc. , so other counter-examples are available	($k = 241$ not required) OR Working mod 17, E1 $36^4 + 1 \equiv 2^4 + 1 = 17 \equiv 0$ so $n = 36$ is also a counter-example (in fact, all $n = 34k + 2$, of course)

Assessment Objectives (AO) Grid

Question	AO1	AO2	AO3(PS)	AO3(M)	Total
1	2	1			3
2	5				5
3	3	2			5
4(i)	1	1	1		3
4(ii)	2				2
4(iii)	2	1			3
4(iv)		1			1
5(i)(a)	1				1
5(i)(b)	1				1
5(i)(c)	4		1		5
5(i)(d)	1				1
5(ii)		1	2		3
6(i)	1	1		1	3
6(ii)	6				6
6(iii)	1		1	1	3
6(iv)				2	2
7(i)(a)	2	3			5
7(i)(b)	1	1			2
7(ii)(a)	1	2			3
7(ii)(b)	2	1			3
Totals	36	15	5	4	60

PS = Problem Solving

M = Modelling

Summary of Updates

Date	Version	Change
October 2019	2	Amendments to the front cover rubric instructions to candidates

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AS Level Further Mathematics A**Y535 Additional Pure Mathematics**

Printed Answer Booklet

Version 2

Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

You must have:

- Question Paper Y535 (inserted)
- Formulae AS Level Further Mathematics A

You may use:

- a scientific or graphical calculator



First name

Last name

Centre
numberCandidate
number**INSTRUCTIONS**

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1	
2	

3

4(i)**4(ii)****(answer space continued on next page)**

4(ii) (continued)	
4(iii)	
4(iv)	
5 (i)(a)	
5(i)(b)	

5(i)(c)	
	5(i)(d)

5 (ii)

6(i)**6(ii)****(answer space continued on next page)**

6(ii) (continued)	
6(iii)	
6(iv)	

7(i)(a)	
	7(i)(b)

7(ii)(a)	
	7(ii)(b)

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