

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Sample Question Paper

Date – Morning/Afternoon

Version 2.1

Time allowed: 1 hour 30 minutes

You must have:

- Printed Answer Booklet

You may use:

- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Formulae

AS Level Mathematics A (H230)

Binomial series

$$(a + b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x - \bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\Sigma f(x - \bar{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$$

The binomial distribution

$$\text{If } X \sim B(n, p) \text{ then } P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \text{ mean of } X \text{ is } np, \text{ variance of } X \text{ is } np(1-p)$$

Kinematics

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

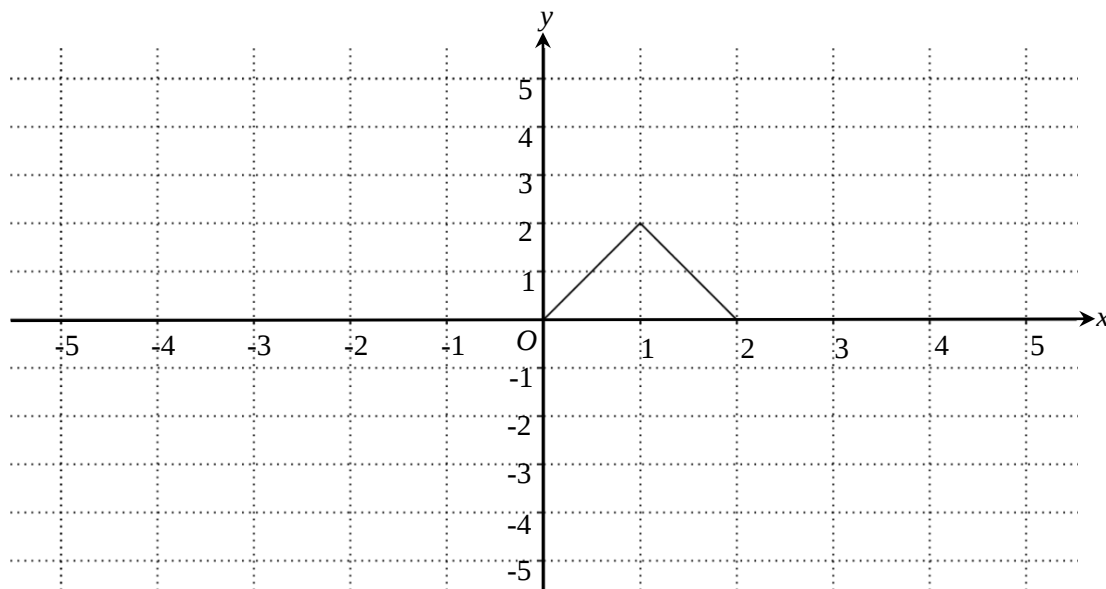
$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Section A: Pure Mathematics

Answer **all** the questions

- 1 (a) The diagram below shows the graph of $y = f(x)$.



- (i) On the diagram in the Printed Answer Booklet draw the graph of $y = f(x+3)$. [2]

- (ii) Describe fully the transformation which transforms the graph of $y = f(x)$ to the graph of $y = -f(x)$. [1]

- (b) The point $(2, 3)$ lies on the graph of $y = g(x)$.

State the coordinates of its image when $y = g(x)$ is transformed to

- (i) $y = 4g(x)$ [1]

- (ii) $y = g(4x)$. [1]

2 In this question you must show detailed reasoning.

Solve the equation $2 \cos^2 x = 2 - \sin x$ for $0^\circ \leq x \leq 180^\circ$. **[5]**

3 The number of members of a social networking site is modelled by $m = 150e^{2t}$, where m is the number of members and t is time in weeks after the launch of the site.

(a) State what this model implies about the relationship between m and the rate of change of m . **[2]**

(b) What is the significance of the integer 150 in the model? **[1]**

(c) Find the week in which the model predicts that the number of members first exceeds 60 000. **[3]**

(d) The social networking site only expects to attract 60 000 members.

Suggest how the model could be refined to take account of this. **[1]**

4 The points A , B and C have position vectors $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$ respectively.
 M is the midpoint of BC .

(a) Find the position vector of the point D such that $\overrightarrow{BC} = \overrightarrow{AD}$. **[3]**

(b) Find the magnitude of $\overrightarrow{AM}$. **[3]**

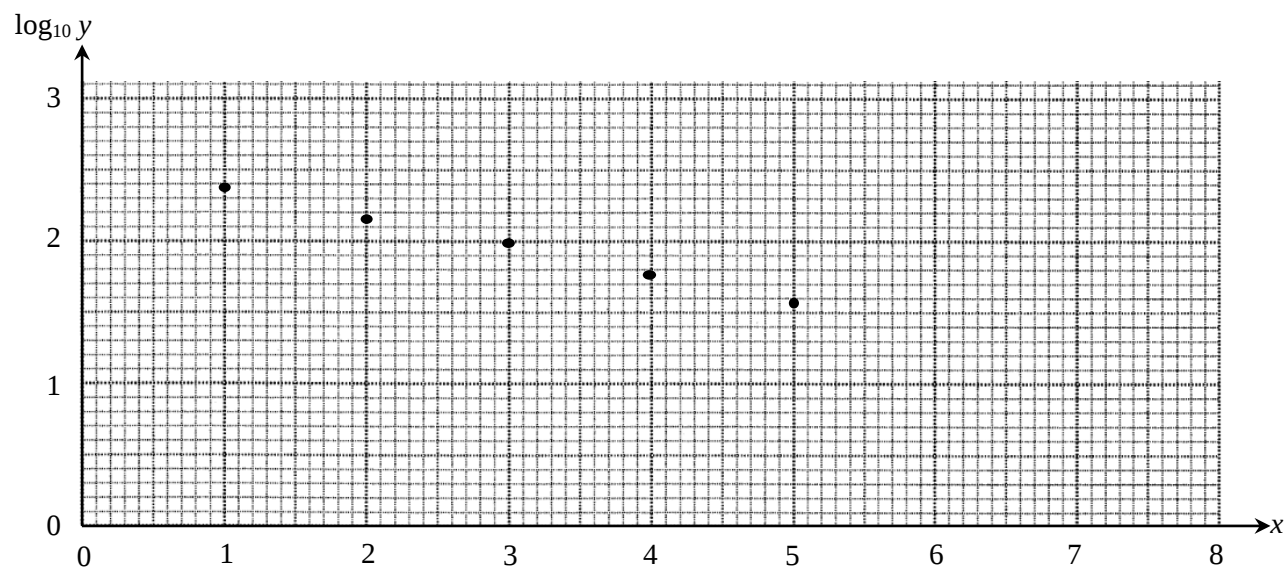
- 5 A doctors' surgery starts a campaign to reduce missed appointments. The number of missed appointments for each of the first five weeks after the start of the campaign is shown below.

Number of weeks after the start (x)	1	2	3	4	5
Number of missed appointments (y)	235	149	99	59	38

This data could be modelled by an equation of the form $y = pq^x$ where p and q are constants.

- (a) Show that this relationship may be expressed in the form $\log_{10} y = mx + c$, expressing m and c in terms of p and/or q . [2]

The diagram below shows $\log_{10} y$ plotted against x , for the given data.



- (b) Estimate the values of p and q . [3]
- (c) Use the model to predict when the number of missed appointments will fall below 20. Explain why this answer may not be reliable. [2]

- 6** **(a)** A student suggests that, for any prime number between 20 and 40, when its digits are squared and then added, the sum is an odd number.

For example, 23 has digits 2 and 3 which gives $2^2 + 3^2 = 13$, which is odd.

Show by counter example that this suggestion is false. **[2]**

- (b)** Prove that the sum of the squares of any three consecutive positive integers cannot be divided by 3. **[3]**

- 7** Differentiate $f(x) = x^4$ from first principles. **[5]**

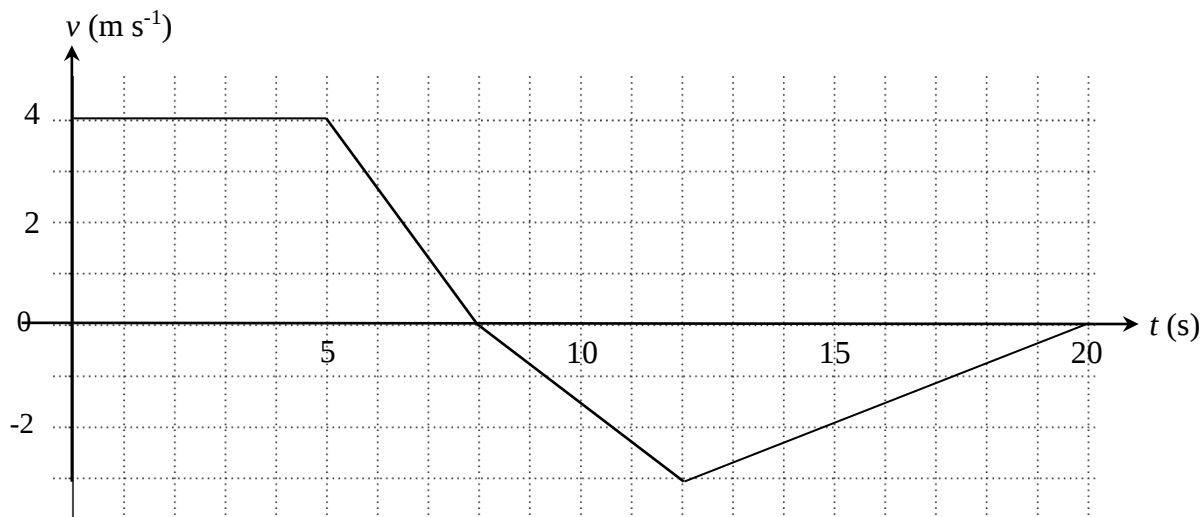
- 8** A curve has equation $y = kx^{\frac{3}{2}}$ where k is a constant.
 The point P on the curve has x -coordinate 4.
 The normal to the curve at P is parallel to the line $2x + 3y = 0$ and meets the x -axis at the point Q .
 The line PQ is the radius of a circle centre P .

Show that $k = \frac{1}{2}$.

Find the equation of the circle. **[10]**

Section B: Mechanics
Answer **all** the questions

- 9 The diagram below shows the velocity-time graph of a car moving along a straight road, where $v \text{ m s}^{-1}$ is the velocity of the car at time $t \text{ s}$ after it passes through the point A.



- (a) Calculate the acceleration of the car at $t = 6$. [2]
- (b) Jasmit says “The distance travelled by the car during the first 20 seconds of the car’s motion is more than five times its displacement from A after the first 20 seconds of the car’s motion”.
Give evidence to support Jasmit’s statement. [3]

- 10 A student is attempting to model the flight of a boomerang. She throws the boomerang from a fixed point O and catches it when it returns to O .

She suggests the model for the displacement, s metres, after t seconds is given by

$$s = 9t^2 - \frac{3}{2}t^3, \quad 0 \leq t \leq 6.$$

For this model,

- (a) determine what happens at $t = 6$, [2]
- (b) find the greatest displacement of the boomerang from O , [4]
- (c) find the velocity of the boomerang 1 second before the student catches it, [2]
- (d) find the acceleration of the boomerang 1 second before the student catches it. [2]

11 In this question the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in the directions east and north respectively.

Distance is measured in metres and time in seconds.

A ship of mass 100 000 kg is being towed by two tug boats.

- The cables attaching each tug to the ship are horizontal.
- One tug produces a force of $(350\mathbf{i} + 400\mathbf{j})$ N.
- The other tug produces a force of $(250\mathbf{i} - 400\mathbf{j})$ N.
- The total resistance to motion is 200 N.
- At the instant when the tugs begin to tow the ship, it is moving east at a speed of 1.5 m s^{-1} .

(a) Explain why the ship continues to move directly east. **[2]**

(b) Find the acceleration of the ship. **[2]**

(c) Find the time which the ship takes to move 400 m while it is being towed.

Find its speed after moving that distance. **[6]**

END OF QUESTION PAPER

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